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Can the Zee ansatz for neutrino masses be correct?

Paul H. Frampton [a,c] and Sheldon L. Glashow [b,c]

[a] Department of Physics and Astronomy, University of North Carolina
Chapel Hill, NC 27559

[b] Lyman Laboratory, Harvard University
Cambridge, MA 02138

[c] School of Physics, Korean Institute for Advanced Study
207-43 Cheongryangri-dong, Dongdaemun-gu, Seoul 130-012, Korea

Working in the framework of three chiral neutrinos with Majorana masses, we investigate a scenario first realized in an explicit model by Zee: that the neutrino mass matrix is strictly off-diagonal in the flavor basis, with all its diagonal entries precisely zero. This CP-conserving ansatz leads to two relations among the three mixing angles $(\theta_1, \theta_2, \theta_3)$ and two squared mass differences. We impose the constraint $|m_3^2 - m_2^2| \gg |m_2^2 - m_1^2|$ to conform with experiment, which requires the θ_i to lie nearby one of four 1-parameter domains in θ-space. We exhibit the implications for solar and atmospheric neutrino oscillations in each of these cases. A unique version of the Zee *ansatz* survives confrontation with experimental data, one which necessarily involves maximal just-so vacuum oscillations of solar neutrinos.

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The minimal standard model involves three chiral neutrino states, but it does not admit renormalizable interactions that can generate neutrino masses. Nevertheless, experimental evidence suggests that both solar and atmospheric neutrinos display flavor oscillations, and hence that neutrinos do have mass. Two very different neutrino squared-mass differences are required to fit the data:

$$10^{-11}\ \mathrm{eV}^2 \le \Delta_s \le 10^{-5}\ \mathrm{eV}^2 \qquad \text{and} \qquad \Delta_a \simeq 10^{-3}\ \mathrm{eV}^2\,, \tag{1}$$

where the neutrino masses m_i are ordered such that:

$$\Delta_s \equiv |m_2^2 - m_1^2| \qquad \text{and} \qquad \Delta_a \equiv |m_3^2 - m_2^2| \simeq |m_3^2 - m_1^2|\,, \tag{2}$$

and the subscripts s and a pertain to solar and atmospheric oscillations respectively. The large uncertainty in Δ_s reflects the several potential explanations of the observed solar neutrino flux: in terms of vacuum oscillations or large-angle or small-angle MSW solutions, but in every case the two independent squared-mass differences must be widely spaced with

$$r \equiv \Delta_s/\Delta_a < 10^{-2}\,. \tag{3}$$

In a three-family scenario, four neutrino mixing parameters suffice to describe neutrino oscillations, akin to the four Kobayashi-Maskawa parameters in the quark sector.[1] Solar neutrinos may exhibit an energy-independent time-averaged suppression due to Δ_a, as well as energy-dependent oscillations depending on Δ_s/E. Atmospheric neutrinos may exhibit oscillations due to Δ_a, but they are almost entirely unaffected by Δ_s. It is convenient to define neutrino mixing angles as follows:

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} c_2c_3 & c_2s_3 & s_2e^{-i\delta}\\ +c_1s_3 + s_1s_2c_3e^{i\delta} & -c_1c_3 - s_1s_2s_3e^{i\delta} & -s_1c_2\\ +s_1s_3 - c_1s_2c_3e^{i\delta} & -s_1c_3 - c_1s_2s_3e^{i\delta} & +c_1c_2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}\,, \tag{4}$$

with s_i and c_i standing for sines and cosines of θ_i. For neutrino masses satisfying (1), the vacuum survival probability of solar neutrinos is [3]:

$$P(\nu_e \to \nu_e)\big|_s \simeq 1 - \frac{\sin^2 2\theta_2}{2} - \cos^4\theta_2\,\sin^2 2\theta_3\,\sin^2(\Delta_s R_s/4E)\,. \tag{5}$$

[1] Two additional convention-independent phases are measurable in principle, but they ordinarily do not affect neutrino oscillations [1] [2].

whereas the transition probabilities of atmospheric neutrinos are:

$$
\begin{aligned}
P(\nu_\mu \to \nu_\tau)\big|_a &\simeq \sin^2 2\theta_1 \cos^4\theta_2 \, \sin^2(\Delta_a R_a/4E)\,,\\
P(\nu_e \leftrightarrow \nu_\mu)\big|_a &\simeq \sin^2 2\theta_2 \sin^2\theta_1 \, \sin^2(\Delta_a R_a/4E)\,,\\
P(\nu_e \to \nu_\tau)\big|_a &\simeq \sin^2 2\theta_2 \cos^2\theta_1 \, \sin^2(\Delta_a R_a/4E)\,.
\end{aligned}
\tag{6}
$$

None of these probabilities depend on δ, the measure of CP violation.

Let us turn to the origin of neutrino masses. Among the many renormalizable and gauge-invariant extensions of the standard model that can do the trick are:

- The introduction of a complex triplet of mesons (T^{++}, T^{+}, T^{0}) coupled bilinearly to pairs of lepton doublets. They must also couple bilinearly to the Higgs doublet(s) so as to avoid spontaneous $B-L$ violation and the appearance of a massless and experimentally excluded majoron. This mechanism can generate an arbitrary complex symmetric Majorana mass matrix for neutrinos.
- The introduction of singlet counterparts to the neutrinos with very large Majorana masses. The interplay between these mass terms and those generated by the Higgs boson—the so-called see-saw mechanism—yields an arbitrary but naturally small Majorana neutrino mass matrix.
- The introduction of a charged singlet meson f^+ coupled antisymmetrically to pairs of lepton doublets, *and* a doubly-charged singlet meson g^{++} coupled bilinearly both to pairs of lepton singlets and to pairs of f-mesons. An arbitrary Majorana neutrino mass matrix is generated in two loops.
- The introduction of a charged singlet meson f^+ coupled antisymmetrically to pairs of lepton doublets *and* (also antisymmetrically) to a pair of Higgs doublets. This simple mechanism was first proposed by Tony Zee [4] and results (at one loop) in a Majorana mass matrix in the flavor basis (e, μ, τ) of a special form:

$$
\mathcal{M} = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}\,. \tag{7}
$$

We focus on the latter scenario. In particular, we adopt the Zee *ansatz* for $\mathcal{M}$ without committing ourselves to the Zee mechanism for its origin. Related discussions of Eq. (7) appear elsewhere [1] [5]. The present work is essentially a continuation of ref. [1].

Because the diagonal entries of $\mathcal{M}$ are zero, the amplitude for no-neutrino double beta decay vanishes at lowest order [4] and this process cannot proceed at an observable

rate. Furthermore, the parameters $m_{e\mu}$, $m_{e\tau}$ and $m_{\mu\tau}$ may be taken real and non-negative without loss of generality, whence $\mathcal{M}$ becomes real as well as traceless and symmetric. With this convention, the analog to the Kobayashi-Maskawa matrix becomes orthogonal: $\mathcal{M}$ is explicitly CP invariant and $\delta = 0$. But as we have noted, it is well known [3] that the mere existence of a squared-mass hierarchy virtually precludes any detectable manifestation of CP violation in the neutrino sector.

The sum of the neutrino masses (the eigenvalues of $\mathcal{M}$) vanishes:

$$m_1 + m_2 + m_3 = 0\,. \tag{8}$$

An important result emerges when the squared-mass hierarchy Eqs. (1) is taken into account along with Eq. (8). In the limit $r \to 0$, two of the squared masses must be equal. There are two possibilities. In case A, we have $m_1 + m_2 = 0$ and $m_3 = 0$. This case arises iff at least one of the three parameters in $\mathcal{M}$ vanishes. In case B, we have $m_1 = m_2$ and $m_3 = -2m_2 > 0$. This case arises iff the three parameters in $\mathcal{M}$ are equal to one another. Of course, r is small but it does not vanish: in neither case can the above relations among neutrino masses be strictly satisfied. But they must be nearly satisfied. Consequently we may deduce certain approximate but useful restrictions on the permissable values of the neutrino mixing angles θ_i. Prior to examining these restrictions, we note that Eqs. (8) and (1) exclude the possibility that the three neutrinos are nearly degenerate in mass. If the Zee *ansatz* is even approximately realized in nature, no neutrino mass can exceed a small fraction of an elecron volt in magnitude and neutrinos are unlikely to contribute significantly to the dark matter of the universe.

We first consider case A. The relation $m_1 + m_2 = 0$ may be obtained in three ways depending on which parameter in $\mathcal{M}$ is set to zero. If $m_{e\mu} = 0$, the quantum number $L_\tau - L_e - L_\mu$ is conserved. It follows that $\cos\theta_1 = 0$ and $\theta_3 = \pi/4$. We see from Eq. (6) that atmospheric ν_μ's oscillate exclusively into ν_e's and *vice versa.* This subcase (or any nearby assignment of mixing angles) is strongly disfavored by SuperKamiokande data [6].

Alternatively we may set $m_{e\tau} = 0$ to obtain conservation of $L_\mu - L_\tau - L_e$. For this subcase, we obtain $\sin\theta_1 = 0$ and $\theta_3 = \pi/4$. We see from Eq. (6) that atmospheric ν_μ's do not oscillate at all. This subcase (or any nearby assignment of mixing angles) is also strongly disfavored by SuperKamiokande data [6].

The last (and best [1]) version of case A has $m_{\mu\tau} = 0$ and leads to conservation of $L_e - L_\mu - L_\tau$. For this subcase, we obtain $\sin\theta_2 = 0$ and $\theta_3 = \pi/4$. We see from Eq. (5) that solar neutrino oscillations are maximal:

$$P(\nu_e \to \nu_e)\big|_s = 1 - \sin^2(\Delta_s R_s/4E)\,. \tag{9}$$

Moreover, we see from Eq. (6) that atmospheric ν_μ's oscillate exclusively into ν_τ's with the unconstrained mixing angle θ_1:

$$\begin{aligned} P(\nu_\mu \to \nu_\tau)\big|_a &= \sin^2 2\theta_1\, \sin^2(\Delta_a R_a/4E)\,, \\ P(\nu_\mu \leftrightarrow \nu_e)\big|_a = 0\,, &\qquad P(\nu_e \to \nu_\tau)\big|_a = 0\,. \end{aligned} \tag{10}$$

This implementation of the Zee *ansatz* is compatible with experiment: It predicts maximal solar oscillations (without an energy-independent term) and it is consistent with the just-so vacuum oscillation hypothesis [7]. However, it is evidently not compatible with the small-angle MSW explanation of solar neutrino data. Neither is it compatible with the large-angle MSW solution, because it predicts virtually maximal solar neutrino oscillations. If $m_{\mu\tau}$ is permitted to depart slightly from zero so as to generate a small finite value of $r \equiv \Delta_s/\Delta_a$, the coefficient of the oscillatory term in Eq. (9) will depart from unity by a term of order $r^2 \leq 10^{-4}$. The resulting solar-neutrino oscillations remain nearly maximal: they are energy independent and experimentally disfavored unless Δ_s lies within the just-so domain.

We furthermore note that the $m_{\mu\tau} \simeq 0$ version of the Zee *ansatz* admits atmospheric neutrino oscillations of the type $\nu_\mu \to \nu_\tau$ with any value of the mixing angle θ_1. At the same time, it precludes all oscillations involving atmospheric ν_e. These results are quite in accord with SuperKamiokande data.

Others have noted [8] that the relation $m_1^2 = m_2^2$ is preserved by radiative corrections for case A. This is not necessarily true for case B, where the relations satisfied by the neutrino masses, $m_3 = -2m_2 = -2m_1$, are not a consequence of a symmetry principle. In any event, we argue that case B cannot fit the data. Ref. [1] shows that case B leads to the relation:

$$\tan^2\theta_2 = 1/2\,. \tag{11}$$

Along with Eqs. (6), this implies:

$$P(\nu_e \to \nu_e)\big|_a = 1 - (8/9)\,\sin^2(\Delta_a R_a/4E)\,. \tag{12}$$

That is, we must have large (almost maximal) oscillations of atmospheric ν_e. This result is strongly disfavored by SuperKamiokande data [6], so that case B can be rejected without further ado.

Our conclusion is simple. We find that one (and only one!) of the realizations of the Zee *ansatz* incorporating a squared-mass hierarchy is compatible with both solar and atmospheric neutrino data. It corresponds to the assignments $m_{e\mu} \simeq m\cos\theta_1$, $m_{e\tau} \simeq m\sin\theta_1$ and $m_{\mu\tau} \ll m$, with $\theta_2 \simeq 0$ and $\theta_3 \simeq \pi/4$. Near this domain, atmospheric electron neutrinos oscillate negligibly, while atmospheric muon neutrinos oscillate into tau neutrinos with the arbitrary mixing angle θ_1. Solar neutrino oscillations are very nearly maximal. They can be described by vacuum oscillations, but not by MSW oscillations. It is straightforward to implement the Zee model so as to conserve $L_e - L_\mu - L_\tau$ *exactly* so as to obtain $m_{\mu\tau} = 0$ to all orders [8]. Of course, this must not be done! Some unspecified new physics (beyond the introduction of Zee's f^+ meson) is required to lift the degeneracy between m_1^2 and m_2^2 so as to yield an extreme hierarchy of neutrino squared-mass differences, with $\Delta_s \sim 10^{-8}\,\Delta_a$.

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